②

Exercise 2.2.6

a. From order properties:

let $x_n < \varepsilon$

then $\frac{1}{n^p} < \varepsilon$

so $n > \varepsilon^{-\frac{1}{p}}$

By Archemedes:

$$(\forall \varepsilon^{-\frac{1}{p}} \in \mathbb{R})(\exists N \in \mathbb{N})[N > \varepsilon^{-\frac{1}{p}}]$$

b. By order properties:

given that $x_N < \varepsilon$

and that $\frac{1}{(n+1)^p} < \frac{1}{n^p}$ for $p > 0$

then $x_n \leq x_N < \varepsilon \quad \forall n \geq N$

c. We have shown that $\langle x_n \rangle_n$ is eventually smaller than all $\varepsilon > 0$.

$$(\forall \varepsilon \in \mathbb{R})(\forall n \in \mathbb{N})(\exists N \in \mathbb{N})[n \geq N \Rightarrow x_n < \varepsilon]$$

which is equivalent to $\lim_{n \to \infty} x^n = 0$